QUANTUM MEASUREMENT WITH IMPERFECT DEVICES ‡

Nicholas Wheeler, Reed College Physics Department
February 2000

Introduction. While Maxwell and others have given attention to the general principles of scientific instrument design, and while there does exist an elaborate theory relating to the "design of experiments" which rewards its students with some very surprising results,[1] there exists, so far as I am aware, no "classical theory of measurement." I suppose the subject has been held to be too obvious, too depended upon contingent circumstance . . . to support such a theory.

Quantum mechanics, on the other hand, has assigned central importance to a distinctive theory of measurement almost from its inception.[2] That very fact has been held up in evidence of how revolutionary was/is the quantum mechanical view of the world.

The quantum theory of measurement—though basic to one of the most successful physical theories ever devised—remains (and is, arguably, today more than ever) deeply problematic . . . which some read as indication that the quantum theory of the future will have abandoned some of the prevailing orthodoxy. But the results I have to report today relate only tangentially if at all to the issues in dispute. I work well within the bounds of orthodox quantum mechanics, but will argue that *the concept of "quantum state" is more ellusive than it is commonly acknowledged to be*. I will show how the canonical quantum theory of measurement (which relates to the idealized operation of perfect devices) can be extended to yield a *theory of imperfect devices*, and how one can *quantify the information gained* by use of such a device.

It best serves my expository purposes to adopt the level of abstraction associated with the name of Dirac. The *state* of a quantum system $\mathfrak{S}$ will—provisionally, at least—be represented by a unit vector $|\psi)$ in a complex inner

‡ Notes for a Reed College Physics Seminar presented 16 February 2000.

[1] See K. S. Banerjee, *Weighing Designs* (1975).

[2] The quantum theory of measurement was subjected to searching analysis in John von Neumann's *Mathematische Grundlagen der Quantenmechanik*, which appeared in 1932 (von Neumann was then twenty-nine years old), but related papers had appeared already several years previously.

product space $\mathcal{H}$, and a *device* (an ideal "A-meter") by a self-adjoint operator A that acts on the elements of $\mathcal{H}$. With each such device can by

$$\begin{aligned}\mathsf{A}|a) &= a|a)\\ &a \text{ ranges on the real-valued } \textit{spectrum} \text{ of } \mathsf{A}\end{aligned}$$

associate an orthonormal basis in $\mathcal{H}$, the "A eigenbasis." To avoid expository complications (but for no deeper reason) we generally assume the spectrum of A to be non-degenerate. The completeness relation

$$\sum\!\!\!\!\!\!\int |a)\,da\,(a| = \mathsf{I}$$

permits arbitrary elements $|\varphi) \in \mathcal{H}$ to be developed

$$|\varphi) = \sum\!\!\!\!\!\!\int |a)\,da\,(a|\varphi)$$

and supplies the "spectral representation" of A:

$$\mathsf{A} = \sum\!\!\!\!\!\!\int |a)\,a\,da\,(a|$$

Continuous spectra (such as should arise in formal representation of a meter stick x) can arise only if $\mathcal{H}$ is ∞-dimensional, and only in the latter case is it possible to realize $[\mathsf{x},\mathsf{p}] = i\hbar\mathsf{I}$. But much of quantum mechanics, and all of the quantum theory of measurement, can be modeled on finite-dimensional state spaces. Particularly transparent are the 2-dimensional models, of which I will make occasional use.

Quantum measurement with perfect devices. One distinguishes in quantum mechanics between two kinds of motion:

- the smoothly continuous *dynamical* motion which is presumed to take place *between* observations, and can (if—arbitrarily—we elect to work in the Schrödinger picture) be described $\mathsf{H}|\psi) = i\hbar\frac{\partial}{\partial t}|\psi)$; with this we will not be much concerned . . . and
- the projectively abrupt/irreversible *state-destruction/reconstruction* which is imagined to be brought about by acts of observation, and will be our principal concern.

The orthodox measurement scenario (which is actually a *state preparation* scenario) runs this way:

System $\mathfrak{S}$, in unknown state $|\psi)_{\text{in}}$, is presented to an A-meter, which announces "a" (one of the eigenvalues of A), signifying that is has abruptly placed $\mathfrak{S}$ in the new state $|\psi)_{\text{out}} = |a)$ (the just-named eigenstate of A). Repetition of the procedure might—owing (not to any instrumental defect, but) to the profoundly statistical nature of the quantum world—have produced a different result. Standard theory asserts, however, that there is pattern in the randomness: that the device can be expected to

$$\begin{aligned}&\text{announce “}a_1\text{” with probability } (\psi|a_1)(a_1|\psi),\\&\text{announce “}a_2\text{” with probability } (\psi|a_2)(a_2|\psi),\\&\qquad\vdots\end{aligned}$$

Standard theory provides no account of *how*—physically, or temporally—the device does what it is alleged to do. The essentials of quantum measurement might therefore be represented

$$|\psi)_{\text{in}} \longrightarrow \boxed{A\text{-meter}} \longrightarrow |\psi)_{\text{out}} = \begin{cases}|a_1) \text{ with probability } |(a_1|\psi)|^2\\|a_2) \text{ with probability } |(a_2|\psi)|^2\\\quad\vdots\\|a_k) \text{ with probability } |(a_k|\psi)|^2\\\quad\vdots\end{cases}$$

which is illustrated in Figure 1.

Suppose that measurement has, in the specific instance, caused the instrument to announce “a_6.” Prompt remeasurement would (owing to the orthonormality of the eigenvectors) yield

$$|a_6)_{\text{in}} \longrightarrow \boxed{A\text{-meter}} \longrightarrow |\psi)_{\text{out}} = \begin{cases}|a_1) \text{ with probability } |(a_1|a_6)|^2=0\\|a_2) \text{ with probability } |(a_2|a_6)|^2=0\\\quad\vdots\\|a_6) \text{ with } \textit{certainty}\\\quad\vdots\end{cases}$$

and it is only on this basis—“confirmation of the prepared state”—that we can claim to have learned something when we performed the first measurement.

The expected average of many such measurements (the presumption here is that we have been supplied with many copies of the object “$\mathfrak{S}$ in state $|\psi)_{\text{in}}$”) becomes

$$\langle \mathbf{A}\rangle_\psi = \sum(\psi|a_k)a_k(a_k|\psi) = (\psi|\mathbf{A}|\psi) \tag{1}$$

Mixed states. Suppose we know of $\mathfrak{S}$ only that it may be

$$\begin{aligned}&\text{in state } |\psi_1) \text{ with probability } p_1,\\&\text{in state } |\psi_2) \text{ with probability } p_2,\\&\qquad\vdots\end{aligned}$$

The average of many measurements then becomes an “average of averages:”

$$\langle \mathbf{A}\rangle = p_1(\psi_1|\mathbf{A}|\psi_1) + p_2(\psi_2|\mathbf{A}|\psi_2) + \cdots$$

Now a little manipulation: use $\mathbf{A} = \sum_n \mathbf{A}|n)(n|$ (where $\big\{|n)\big\}$ refers to any orthonormal basis) to write

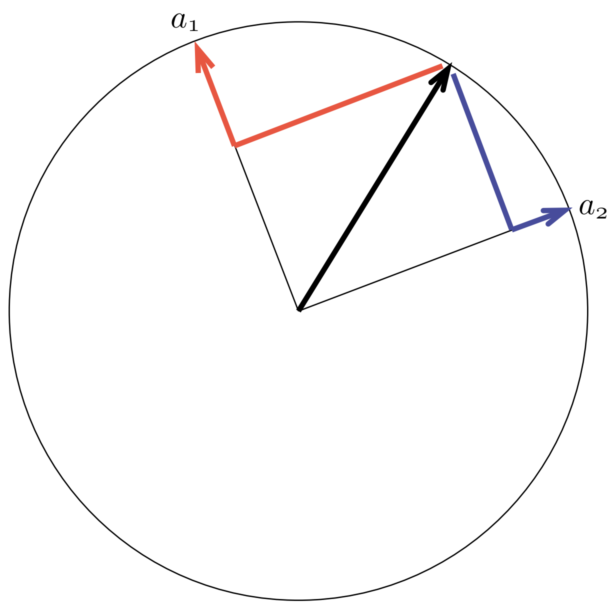

FIGURE 1: *Illustration of the sense in which quantum measurement is "projective." The device makes its own uncontrollable decision whether to perform the red action* $|\psi) \to |a_1)$*, which it does with probability determined by the size* $(a_1|\psi)$ *component in*

$$|\psi) = |a_1)(a_1|\psi) + |a_2)(a_2|\psi) + \cdots$$

or the blue action $|\psi) \to |a_2)$*, or ...*

$$\begin{aligned}\langle \mathbf{A} \rangle &= \sum_k \sum_n p_k(\psi_k|\mathbf{A}|n)(n|\psi_k) \\ &= \sum_n (n|\boldsymbol{\rho}\mathbf{A}|n) \quad \text{with} \quad \boldsymbol{\rho} \equiv \sum_k |\psi_k)p_k(\psi_k| \\ &= \mathrm{tr}\boldsymbol{\rho}\mathbf{A} \end{aligned} \tag{2}$$

The linear operator $\boldsymbol{\rho}$ is an elegantly efficient *descriptor of the mixture*, and is called the "density operator."

If the mixture contains in fact only a single state $|\psi)$ then

$$\boldsymbol{\rho} = |\psi)(\psi| \quad : \quad \text{"pure case"}$$

which is obviously (and, as will emerge, distinctively) projective: $\boldsymbol{\rho}^2 = \boldsymbol{\rho}$. But in all cases (whether "pure" or "mixed") we have

$$\mathrm{tr}\,\boldsymbol{\rho} = \sum_n \sum_k (n|\psi_k)p_k(\psi_k|n) = \sum_k p_k(\psi_k|\psi_k) = \sum_k p_k = 1$$

The density operator, since manifestly self-adjoint, possesses its own set of real eigenvalues and orthonormal eigenvectors $\boldsymbol{\rho}\,|\rho_i) = \rho_i|\rho_i)$, in terms of which it can be described

$$\boldsymbol{\rho} = \sum_i |\rho_i)\rho_i(\rho_i| \tag{3}$$

Orthonormality now entails $\boldsymbol{\rho}^2 = \sum_i |\rho_i)\rho_i^2(\rho_i|$, so we will have projectivity if and only if $\rho_i^2 = \rho_i$ (all i), which by $\mathrm{tr}\,\boldsymbol{\rho} = \sum \rho_i = 1$ forces one eigenvalue to be unity and all others to vanish ... in which case $\boldsymbol{\rho}$ refers to a pure state (as claimed above). In general, one can state that

> density operators are self-adjoint linear operators, distinguished from others in this regard: their eigenvalues are non-negative, and sum to unity.

Of more immediate interest is this fact: the density operator $\boldsymbol{\rho}$, which came to us as a p_k-weighted mixture of states $|\psi_k)$, is in (3) displayed as a ρ_i-weighted mixture of states $|\rho_i)$. *What came to us as a box of mixed nuts and raisins has become a box of mixed persimmons and kumquats*! Which is it ... *really*? The question is fair in the classical world of our daily experience, but is quantum mechanically meaningless. No observation can reveal the "inner constitution" of a mixed state; $\boldsymbol{\rho}$ says all that can be said concerning the state of $\mathfrak{S}$. Mixed states (announced $\boldsymbol{\rho}^2 \neq \boldsymbol{\rho}$) are things-in-themselves: quantum states that happen not to be pure, the name we give to *populations of observationally equivalent mixtures*.

Tom Wieting has devised[3] an elegant way to comprehend the extent of such a population when the state space $\mathcal{H}$ is 2-dimensional, and self-adjoint operators representable therefore by 2×2 Hermitian matrices. In that setting, projection operators $|\psi)(\psi|$ become projection *matrices*, and can be shown to have the form

$$\mathbb{P} = \tfrac{1}{2}\big\{\mathbb{I} + \psi_1\boldsymbol{\sigma}_1 + \psi_2\boldsymbol{\sigma}_2 + \psi_3\boldsymbol{\sigma}_3\big\}$$

where

$$\boldsymbol{\psi} = \begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix} \quad \text{is a real unit 3-vector}$$

and the $\boldsymbol{\sigma}_i$ are Pauli matrices. One can thus associate states $|\psi)$ with points $\boldsymbol{\psi}$ on the unit 3-sphere. The state $|\psi)_\perp$ orthogonal to $|\psi)$ is in this representation associated with the diametric point; i.e., with the 2-vector upon which

$$\mathbb{P}_\perp = \tfrac{1}{2}\big\{\mathbb{I} - \psi_1\boldsymbol{\sigma}_1 - \psi_2\boldsymbol{\sigma}_2 - \psi_3\boldsymbol{\sigma}_3\big\}$$

---

[3] At 2:55 pm on 5 May 1998, between one Senior Oral and the next. He had an instant off-the-top-of-the-head response when I happened to mention my interest in the problem, and by 5:00 pm (when I chanced to meet him emerging from his next examination) had a written account of the details.

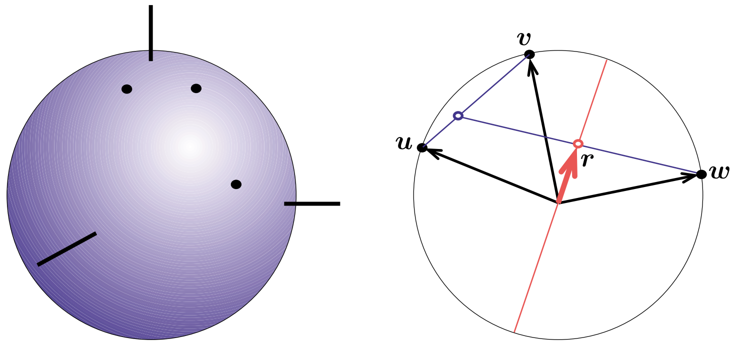

FIGURE 1: *At left, three weighted points on the unit 3-ball represent a mixture of three quantum states. On the right a dimension has been discarded: the unit 3-ball has become the unit circle, on which weighted points* $\{\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}\}$ *are deposited. Constructions indicate how one might compute the center of mass of* $\{\boldsymbol{u},\boldsymbol{v}\}$*, then of* $\{\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}\}$ *to determine finally the location* $\boldsymbol{r}$ *of the "center of mass of the mixture." Eigenstates of the density operator are associated with points where the red diameter contacts the sphere.*

projects. Density operators acquire representation as populations of weighted points $\{\boldsymbol{\psi}_1$ of weight $p_1, \boldsymbol{\psi}_2$ of weight $p_2,\dots\}$ sprinkled on the surface of the 3-sphere. "Equivalent populations" are populations which share the same center of mass. The mixture will be pure or impure according as the center of mass lies on the surface of the sphere, or in its interior. Figure 2 illustrates the essential idea. Thus far neither Wieting nor I have been able either to devise or to discover in the literature a workable higher-dimensional generalization of what I call "Wieting's construction."[4]

Presentation of a mixed state to a perfect device. The abrupt process in question can be depicted

$$\boldsymbol{\rho}_{\text{in}} = \sum_i |\rho_i)\rho_i(\rho_i| \longrightarrow \boxed{A\text{-meter}} \longrightarrow \boldsymbol{\rho}_{\text{out}} = \begin{cases} \boldsymbol{\rho}(a_1) \equiv |a_1)(a_1| & \text{else} \\ \boldsymbol{\rho}(a_2) \equiv |a_2)(a_2| & \text{else} \\ \qquad\vdots & \\ \boldsymbol{\rho}(a_k) \equiv |a_k)(a_k| & \text{else} \\ \qquad\vdots & \end{cases}$$

where evidently

$$\begin{aligned} \text{probability that meter announces “}a_0\text{”} &= \sum_i (a_0|\rho_i)\rho_i(\rho_i|a_0) \\ &= (a_0|\,\boldsymbol{\rho}_{\text{in}}|a_0) \\ &= \text{tr}\big\{\boldsymbol{\rho}_{\text{in}}\,\boldsymbol{\rho}(a_0)\big\} \end{aligned} \tag{4}$$

[4] But see what F. J. Belinfante, in §3.6 of *A Survey of Hidden-Variable Theories* (1973) has to say about A. M. Gleason's concept of "frame function."

Observe that the

$$\text{sum of those probabilities} = \int (a_0|\,\boldsymbol{\rho}_{\text{in}}|a_0)\,da_0 = \text{tr}\,\boldsymbol{\rho}_{\text{in}} = 1 \tag{5}$$

In some applications it proves useful to notice when the meter announces "a_0" the prepared state can be described

$$\boldsymbol{\rho}_{\text{out}}(a_0) = \frac{|a_0)(a_0|\boldsymbol{\rho}_{\text{in}}|a_0)(a_0|}{(a_0|\boldsymbol{\rho}_{\text{in}}|a_0)} = \frac{|a_0)(a_0|\boldsymbol{\rho}_{\text{in}}|a_0)(a_0|}{\text{tr}\big\{\text{numerator}\big\}}$$

where the final equality follows from $\text{tr}\,\mathbf{AB} = \text{tr}\,\mathbf{BA}$ and the fact that $|a_0)(a_0|$ is projective:

$$\text{tr}\big\{\text{numerator}\big\} = \text{tr}\big\{\boldsymbol{\rho}_{\text{in}}|a_0)(a_0|\cdot|a_0)(a_0|\big\} = \text{tr}\big\{\boldsymbol{\rho}_{\text{in}}|a_0)(a_0|\big\} = (a_0|\,\boldsymbol{\rho}_{\text{in}}|a_0)$$

Imperfect devices. One can, on the one hand, imagine an A-meter which, when it constructs $|a_0)$, announces with probability $p(a;a_0)$ that it has constructed $|a)$. One can equally well imagine a meter which, when it announces "a_0", has with probability $p(a_0;a)$ actually constructed $|a)$. Or a meter which is faulty in both respects. I believe it to be the case—but can today supply no definitive proof—that all modes of imperfection are functionally equivalent, that I have been describing what is—quantum mechanically, if not classically—a "distinction without a difference." In any event...

I will explore implications of the proposition that *all meters prepare named mixtures* (which in the idealized case of a "perfect" meter will be "pure"):

$$\boldsymbol{\rho}_{\text{in}} \longrightarrow \boxed{A\text{-meter announces "}a_0\text{"}} \longrightarrow \boldsymbol{\rho}_{\text{out}} = \boldsymbol{\rho}(a_0)$$

where

$$\boldsymbol{\rho}(a_0) \equiv \int |a)(a|\cdot p(a_0;a)\,da \tag{6}$$

becomes "pure" in the case $p(a_0;a) = \delta(a - a_0)$. Evidently the *probability* that the imperfect meter will, upon examination of $\boldsymbol{\rho}_{\text{in}}$, announce"$a_0$" can be described

$$\begin{aligned}\text{probability of "}a_0\text{"} &= Z^{-1}\cdot\int (a|\boldsymbol{\rho}_{\text{in}}|a)p(a_0;a)\,da\\ &= Z^{-1}\cdot\text{tr}\big\{\boldsymbol{\rho}_{\text{in}}\,\boldsymbol{\rho}(a_0)\big\} \qquad (7)\\ &\downarrow\\ &= (a_0|\boldsymbol{\rho}_{\text{in}}|a_0)\ \text{as device becomes perfect}\end{aligned}$$

Here

$$Z \equiv \int \text{tr}\big\{\boldsymbol{\rho}_{\text{in}}\,\boldsymbol{\rho}(a_0)\big\}\,da_0 \tag{8}$$

is a normalization factor, introduced to insure that

$$\int (\text{probability of "}a_0\text{"})\, da_0 = 1$$

For perfect meters the argument which gave (5) gives

$$Z = 1 \quad : \quad (\text{all } \boldsymbol{\rho}_{\text{in}})$$

but for imperfect meters the value of Z is contingent upon $\boldsymbol{\rho}_{\text{in}}$, and might more properly be denoted $Z(\boldsymbol{\rho}_{\text{in}})$. Were it otherwise, Z could be absorbed into the definition of $p(a_0; a)$ which, I must emphasize, has the character not of a joint distribution function, but of an a_0-parameterized family of a-distributions: we demand

$$\int p(a_0; a)\, da = 1 \quad : \quad (\text{all } a_0)$$

and might expect to have

$$\langle a \rangle = \int p(a_0; a)\, a\, da \sim a_0$$

but have no grounds on which to require $\iint p(a_0; a)\, da_0 da = 1$.

The expected mean of the results of many such measurements becomes

$$\langle \mathsf{A} \rangle = Z^{-1} \cdot \iint (a|\boldsymbol{\rho}_{\text{in}}|a) a_0 p(a_0; a)\, da da_0 \tag{9}$$

In the important class of what I will call "symmetrically transitive" cases, in which $p(a_0; a)$ depends on its arguments only through the square of their difference, as exemplified by

$$p(a_0; a) = \tfrac{1}{\sigma\sqrt{2\pi}} \exp\left\{ \left(\tfrac{a-a_0}{\sigma}\right)^2 \right\}$$

the preceding result simplifies a bit: we have

$$\begin{aligned} \langle \mathsf{A} \rangle &= Z^{-1} \cdot \iint (a|\boldsymbol{\rho}_{\text{in}}|a)\, a\, p(a_0; a)\, da da_0 \\ &= Z^{-1} \cdot \int \text{tr}\big\{ \boldsymbol{\rho}_{\text{in}} \mathsf{A}\, \boldsymbol{\rho}(a_0) \big\}\, da_0 \qquad (10) \\ &\downarrow \\ &= \text{tr}\big\{ \boldsymbol{\rho}_{\text{in}} \mathsf{A} \big\} \text{ as device becomes perfect} \end{aligned}$$

Prompt remeasurement, after an imperfect device has responded "a_0" will *not* reproduce "a_0" with certainty; it will produce "a_1" with

$$\begin{aligned} \text{probability of "}a_1\text{" after "}a_0\text{"} &= Z^{-1} \cdot \text{tr}\big\{ \boldsymbol{\rho}(a_0) \boldsymbol{\rho}(a_1) \big\} \qquad (11.1) \\ &\downarrow \\ &= \delta(a_1 - a_0) \text{ as device becomes perfect} \end{aligned}$$

where

$$Z = Z(a_0) \equiv \int \text{tr}\big\{ \boldsymbol{\rho}(a_0) \boldsymbol{\rho}(a_1) \big\}\, da_1 \tag{11.2}$$

This result conforms very neatly to what we intuitively mean when we say of a measurement device that it operates "imperfectly."

Entropy of quantum states, information gained by measurement. Let

$$\mathcal{P} \equiv \{p_1, p_2, \ldots, p_n\} \quad : \quad \sum p_i = 1$$

be a set of non-negative real numbers which sum to unity, such as might be assigned to a random variable which can assume one or another of n mutually exclusive values. It was, I believe, Boltzmann who first drew attention to the entropy-like properties of the construction

$$S(\mathcal{P}) \equiv -\langle \log p \rangle = -\sum_{i=1}^{n} p_i \log p_i \tag{12}$$

which today is taken to *define* the "entropy of the set $\mathcal{P}$." One can show that

$$0 \leqslant S(\mathcal{P}) \leqslant n \log n$$

with

$$S(\mathcal{P}) = \begin{cases} 0 & \text{if and only if one } p_i = 1 \text{ and the rest vanish} \\ n \log n & \text{if and only if all } p_i \text{ are the same} \end{cases}$$

So $S(\mathcal{P})$ increases as $\mathcal{P}$ becomes more undifferentiated/grey/uniform.

von Neumann, who had in mind applications to the statistical mechanics of quantum systems—mixtures of the thermalized form

$$\boldsymbol{\rho} = \sum_n |n)e^{-E_n/kT}(n| \quad \text{with} \quad \mathbf{H}|n) = E_n|n)$$

—noticed that one can, quite generally, assign entropy to mixed states: let $\boldsymbol{\rho}$ be presented in spectral form

$$\boldsymbol{\rho} = \sum_i |\rho_i)\rho_i(\rho_i|$$

Then von Neumann would have us write

$$S(\boldsymbol{\rho}) \equiv -\sum_i \rho_i \log \rho_i = -\text{tr}\{\boldsymbol{\rho} \log \boldsymbol{\rho}\} \tag{13}$$

(where one must work a little bit to give precise meaning to $\log \boldsymbol{\rho}$).

Claude Shannon had interest in *procedures which lead one to adjust* the values of the p_i, and demonstrated the utility of writing

$$\begin{aligned} \text{"information gained" by such a procedure} &\equiv -(\text{entropy lost}) \\ &= S(\mathcal{P}_\text{before}) - S(\mathcal{P}_\text{after}) \end{aligned}$$

We can borrow this idea to lend quantitative meaning to the "information gained by a quantum measurement process:"

$$\text{information gained} = S(\boldsymbol{\rho}_{\text{in}}) - S(\boldsymbol{\rho}_{\text{out}}) \tag{14}$$

We note in this connection that

$$S(\boldsymbol{\rho}_{\text{impure}}) > S(\boldsymbol{\rho}_{\text{pure}}) = 0$$

Perfect devices prepare pure states; for them one has

$$\text{information gained with perfect device} = S(\boldsymbol{\rho}_{\text{in}})$$

The states prepared by imperfect devices are, on the other hand, mixed, so we are brought to the satisfying conclusion that

$$\begin{aligned}\text{information gained with imperfect device} &= S(\boldsymbol{\rho}_{\text{in}}) - S(\boldsymbol{\rho}_{\text{out}})\\ &< S(\boldsymbol{\rho}_{\text{in}}) = \text{information gained with perfect device}\end{aligned}$$

It is entirely possible for the "information gained" by operation of an imperfect device to be *negative*. This will, in fact, be invariably the case when $\boldsymbol{\rho}_{\text{in}}$ is pure. In optics one encounters a similar situation: 100% polarized light, when presented to an imperfect polarizer, will emerge less polarized than it was, and if presented to a perfect depolarizer will emerge completely unpolarized (i.e., with "maximal entropy," as the term is used by opticians).

Conclusions, loose ends & prospects. I recognize a distinction between *counting* and *measuring* (though often we assign "counting numbers" to things we could not actually count, like the number of atoms in this crystal), and am prepared to grant that counting can, at least in simple instances, be done with utter precision: I count the dimensions of physical space, and have absolute confidence in the accuracy of my "3" (but less confidence that is a number of interest to God). But *measurement is invariably imprecise* . . . in point of practical fact, but also, I think, for reasons of deep principle: Nature conspires against its precise quantification.

If those remarks are sound, then every <u>measurement device must of necessity be imperfect</u>; the perfect devices contemplated in the orthodox quantum theory of measurement—represented

$$\text{perfect device} \quad \longleftrightarrow \quad \mathsf{A}$$

—are unphysical idealizations. Realistic devices, by the present account, require more complex description

$$\text{imperfect device} \quad \longleftrightarrow \quad \big\{\mathsf{A},\, p(a_0;a)\big\}$$

and give rise to relatively more complicated expressions, but expressions which do at least appear to be of *manageable* complexity.

It would be of interest (and should not be difficult) to develop a population of concrete *examples* which illustrate the effects of finite instrumental resolution.[5] It should also be possible (on the assumption that the space of states is finite dimensional) to run *numerical simulations* of the action of an imperfect device.

One would like to know what can be said of *moments and correlations* when one works from imperfect data (for example: how much $\Delta x \Delta p$ gets "fuzzed up") and whether clever experimental design might in principle permit one to "strip away the fuzz."

If devices are inherently imperfect, then so also, in particular, must be the energy devices H. But if H exhibits imperfections when used to *measure* energy, then perhaps it should do so also when pressed into service as the generator of quantum dynamical motion. Down this road appears to lie a "fuzzy quantum dynamics." I must, however, stress that in orthodox quantum mechanics the state-adjustments that result from

- unobserved quantum dynamical motion
- observational acts

differ profoundly: the former are continuous/unitary/isentropic, while the latter are abrupt/projective ... and entail information gain/loss.

It seems to me plausible, though I am by no means expert in these fields, that ideas advanced here may bear on the "limits of quantum computation," and that they may lend quantum mechanical relevance to the elaborately developed theory of error correcting codes.

While the little "theory of imperfect quantum measurement" sketched above might (in my view) be held to be intuitively/formally quite satisfying, I must stress that while theories may impeach themselves they cannot validate themselves, and that the question *Does the proposed theory conform to the observed facts of the matter?* remains open. We have interest, therefore, in the results of experiments designed to expose its defects (if any).

---

[5] See LECTURES ON QUANTUM MECHANICS (2000), *Chapter 1: Two-state Systems*, pp. 14–17 for the example which inspired this whole exercise.